EVALUATING THE LEVEL OF TECHNOLOGY DEVELOPMENT – A MATHEMATICAL MODEL

Wacław GIERULSKI, Bożena KACZMARSKA

Abstract: The paper presents the concept of using a mathematical model constructed on the basis of a differential equation for determining technology level as a measure of the organization innovation. This methodology is an attempt to transfer the experience of constructing mathematical models in physics and biology into production engineering within the areas of evaluation and development of innovation. Sample research results for several organizations are presented. However, they should be treated only as a research methodology presentation and not as a credible assessment.

Key words: mathematical model, technology level, innovation

1. Introduction

Mathematical modelling is a well-known and commonly used research tool. Initially it was applied in physics, in particular to activities connected with engineering- mechanics and electrotechnics. Subsequently, mathematical modelling included biology problems (the end of the 19th and the first half of the 20th centuries)following the models applied to physics. Chemistry, biochemistry and medicine are the next areas of science which use mathematical modelling as a research tool. Social sciences and among them economics is the latest area which makes use of mathematical models in the research process. [1, 7, 8]

The development of computer techniques made the construction of mathematical models easier. Specialist mathematical packages allow for acquiring easily, in the form of computer simulations, statistical and dynamic solutions resulting from the model. According to the definition presented by R. Aris [5]: "*Mathematical model is an arbitrary, complete and consistent system of equations, corresponding to a certain quantity, its prototype. The prototype can be a physical, biological, social, psychological or conceptual quantity or even another mathematical model. Mathematical modelling can also be related to social issues including technology development*."

In the process of building a mathematical model one can differentiate two stages. One stage is a descriptive representation of the modelled fragment of reality(processes, phenomena) directed towards distinguishing features that a model should have (heuristic modelling). The other stage is the determination of the mathematical structure relevant to the heuristic model. Mathematical models connected with various areas of science often bear a close resemblance between the applied mathematical structures, which proves the analogies between the described processes and phenomena. In the process of modelling the analogies allow for following the patterns of the already existing models. The primary problem in the construction of mathematical models is the selection of measurable parameters describing the modelled process or phenomenon and ascribing to them a mathematical structure simple enough so it is possible to conduct an analysis with the application of effective methods. Another parameter is the verification of the model linked to the identification of the value of parameters so that the model indicates sufficient similarity to the modelled reality. Sometimes qualitative verification is satisfactory without identifying the value of parameters. The process of parameters selection and verification is easier in the models used in physics when one can conduct and repeat the designed experiments. It is more difficult in models applied in biology, where frequently one can only observe processes occurring in nature without the possibility of conducting repeatable experiments. The largest problem appears in economics and social sciences, where the dominant influence on the occurring processes has the human factor, which obviously does not easily yield to the principles and rules. If the model is to describe the changes then the differential and difference equations are the proper mathematical structure. The differential equations constitute a continuous description, which allows for obtaining a solution for an optionally small argument changes whereas the difference equations constitute a discrete description, in which solutions are determined for abrupt changes. The models of this type belong to the deterministic ones (without the elements of randomness), which means that the changes are unambiguously defined and the determined state depends only on the previous states. Stochastic models (including the econometric ones) have a special role in the description of reality. They use data concerning large populations and often it is difficult relate the conclusions resulting from them to a single element. A large effectiveness of these models is manifested in particular in social and economic sciences where the reality is created through human activities.

2. Technology development level

The works [2,3,4] present an original methodology for evaluating and classifying the organizations such as centres for innovation and entrepreneurship. The methodology can be extended to other types of organizations and in particular to manufacturing enterprises. In accordance with this methodology a map in the form of a matrix of states is created. The states are defined by two parameters: technology level and innovative environment development (Figure 1).

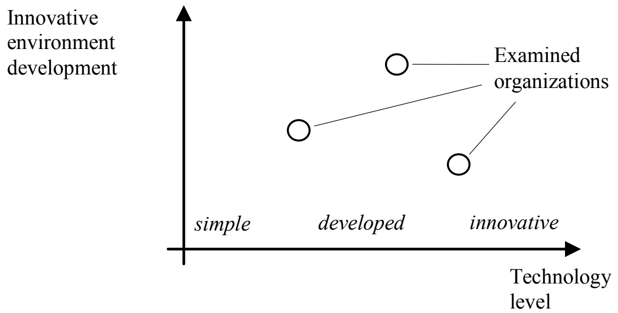

Fig. 1. Map in the form of a matrix of states.
Source: the authors.

The technology level in organizations, from simple through developed to innovative ones, is determined on the horizontal axis. The technology level of the examined Technology Parks was determined in works [2,4] in a discrete way with the application of relational conditions. The result was the assignment to one of the three development stages. A continuous measurement of the technology level calculated with the application of the DEA method was introduced in work [3]. The results obtained were of static nature, they showed only the present state without the possibility of diagnosing the feasibility of changes.

The parameter "technology level" should be understood in a general way and should account for many factors such as, for example: innovation level of products and manufacturing processes, conducting own research within the organization or a close cooperation with external research and science centres, technology transfer, and commercialization of research results, patents, etc. The selection of factors in the technology level evaluation constitutes a separate task dependent on the studied organizations and a detailed purpose of the research.

2. The concept of the technology development model

The concept of the technology development model in the organization is based on models which for a long time have been applied to physics (elementary gas particles collisions) and biology (population growth - the Verhulst model) described with a logistic equation [1,7,8]. The level of technology was assumed as the modelled quantity (in accordance with the previously presented characterization). It is dependent on the variable representing changes. Hence the model should determine the dependency:

$$PT = f(x), \tag{1}$$

where: PT – technology level, x – variable representing changes (argument of a function).

Assuming that the growth rate of technology level is proportional to the technology level (k-proportionality coefficient) we receive a model in the form of a differential equation:

$$\frac{d}{dx}(PT) = k(PT). \tag{2}$$

The solution to this equation is an exponential function in the form of:

$$PT = (PT)_0\, e^{kx}, \tag{3}$$

where $(PT)_0$ is the technology level for $x=0$.

According to the solution the (PT) values reach high values for large values of x, ultimately approaching infinity. The representation is not compatible with reality, which enforces the need for model revision. The technology level growth is a result of introduced changes, which bring positive effects. In the case of high technology level there is less and less space for this type alterations, which can be included in the model (2) by attaching an additional component:

$$\frac{d}{dx}(PT) = k(PT) - b(PT)^2, \tag{4}$$

where the coefficient *b* determines the failure level while introducing developmental changes.

It is possible to apply another line of thought to the process of formulating the heuristic model. The proportionality coefficient k defines the implemented changes. It is the difference between the recommended (invented) changes to be implemented and the number of changes rejected. Assuming that the number of rejected changes is proportional to the technology level, the coefficient k can be written as a mathematical dependence:

$$k = k_1 + b_1(PT), \tag{5}$$

hence the equation (2) takes the form:

$$\frac{d}{dx}(PT) = (k_1 + b_1(PT))(PT), \tag{6}$$

thus after the transformation:

$$\frac{d}{dx}(PT) = k_1(PT) - b_1(PT)^2, \tag{7}$$

Therefore one receives the same mathematical model after substituting $k_1 = k$, and $b_1 = b$. The equations (4) and (7) can be written introducing a different derivative designation:

$$(PT)' = k(PT) - b(PT)^2. \tag{8}$$

On the basis of dependency (8) it is possible to plot a graph $(PT)' = f(PT)$ which shows stationary solutions (Figure 2).

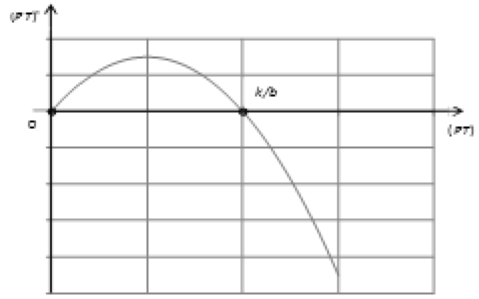

Fig. 2. Stationary solutions of the equation.
Source: the authors.

The differential equation (4) has stationary solutions for x=0, and $x = \frac{k}{b}$. For (*PT*) < *k/b* the derivative is positive that is the function (*PT*) is ascending. For (*PT*) > *k/b* the derivative is negative, which means that the function (*PT*) is descending. The differential equation constituting the model of technology development (8) can be solved analytically.

$$(PT) = \frac{ke^{kx}}{C+be^{kx}}\,. \tag{9}$$

The solution form (Figure 3) depends on the value of parameters *k, b* and the initial value of $(PT)_0$. The integration constant *C* in this solution depends on the initial condition $(PT)_0$:

$$C = \frac{k}{(PT)_0} - b\,. \tag{10}$$

The function $f_1(x)$ corresponds to small values of the initial condition and represents a typical logistic function. During the initial stage the increase (*PT*) is slow, it requires a significant change of the argument x. In the middle stage the growth (*PT*) is faster and a significant slowdown appears in the final stage. The boundary value is the asymptote (*PT*) = *k/b* which is approached by the function value (stationary solution). The function *c* corresponds to higher values of the initial condition and has a shape close to an exponential function. The asymptote (*PT*) = *k/b* is the boundary function. The function $f_3(x)$ corresponds to large values of the initial condition, its shape is close to an exponential function and illustrates diminishing of *(PT)* to the boundary value *k/b*.

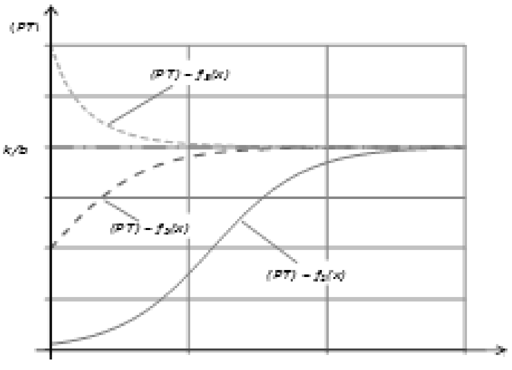

Fig. 3. Equation solutions for different initial conditions
Source: the authors

Only the logistic function $f_1(x)$ can be used for modelling the technology development. It illustrates growth (resulting from development) and describes this process starting with any small initial values. The growth rate is determined by the derivative of function (*PT*) after argument x, which is represented by the following dependency:

$$(PT)' = \frac{Ck^2e^{kx}}{(C+be^{kx})^2}\,. \tag{11}$$

Figure 4 illustrates the logistic function assumed as a model of technology development, and its derivative demonstrating the rate of changes. The rate of changes, in accordance with the function derivative, determines how the increment of the function argument affects the change of the function value (in other words susceptibility to change). Apart from the information on the current state of the technology level this approach allows for determining an additional feature of the examined organization, namely the effort required for raising this state.

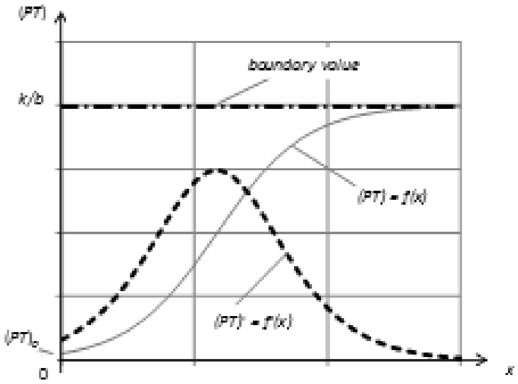

Fig. 4. Technology development model
Source: the authors

3. Technology level measure

Defining the measures assigned to variables linked to the coordinates system is another element of the model structure. The axis (PT) determines the level of technology from the lowest to the highest one. The lowest value can be close to 0 and depends on the initial values. The highest value should correspond to the most advanced technologies. Assuming the changeability range is a matter of pre-arrangement, and it can be represented by numbers or percentages (0%-100%) after relating to the maximum value. Any other range can be brought to these values. It is justifiable to assign descriptive definitions corresponding to different ranges of values (PT). Technologies close to high values can be called innovative. The opposite may be the technologies described as conservative (non-innovative). The conservatism level and the innovation level can be graded, which allows us to separate zones on the axis (PT) (Figure 5). These zones can include different changeability intervals (PT).

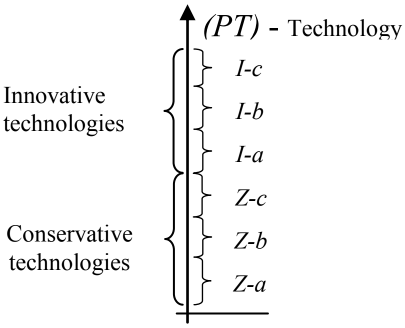

Fig. 5. Technology level intervals.
Source: the authors.

Names and symbols assigned to the zones are presented in Table 1.

Table 1

Innovation levels			
Conservative technologies		Innovative Technologies	
definitely conservative	Z – a	little innovative	I – a
medium –conservative	Z – b	medium-innovative	I – b
moderately conservative	Z – c	definitely innovative	I - c

Source: the authors.

The continuous measure and gradation of the technology level will make it possible to avoid a two-state evaluation (innovative-non-innovative) in the research process, which describes the actual state in a more precise way. Determining the argument measure of the logistic function is a major problem. Models applied to physics and biology use time as the argument, and all the changes naturally relate to the flow of time. It is a specific variable, taking only non-negative values and growing at a constant rate independent of human activities. Assuming time as a uniform logistic function argument in the technology development model is possible, however the rate of implemented developmental changes depends on human decisions and it is not constant. It is necessary then to apply an individual logistic function to descriptions of development of each organization. It would be possible due to an analysis of historical data including long-standing periods (Figure 6).

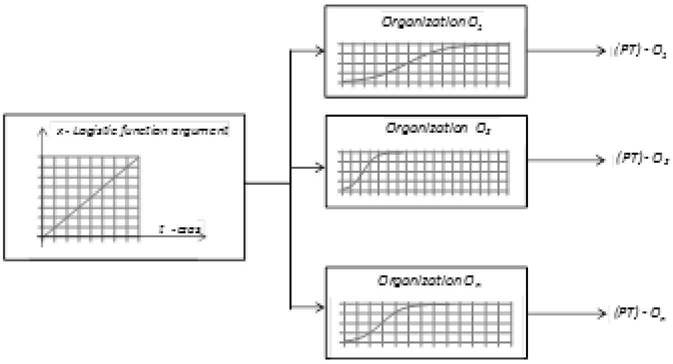

Fig. 6. Uniform arguments of logistic function.
Source: the authors.

It is possible to reverse the problem, assuming one universal model (logistic function) and determine the increment of the argument individually for each of the examined organizations. Especially selected development factors (e.g. indicated in Chapter 1), assigned to the studied organization aggregated to one parameter constituting the argument (x) of the logistic function will be the measure of a variable understood in this way. These factors (attributes) change over time according to an unknown, unidentified, and in the majority of cases non-linear dependency different for each organization (Figure 7), which results from a distinct rate of changes. The values of the argument (x) can also diminish, which does not occur in the case of a variable such as time.

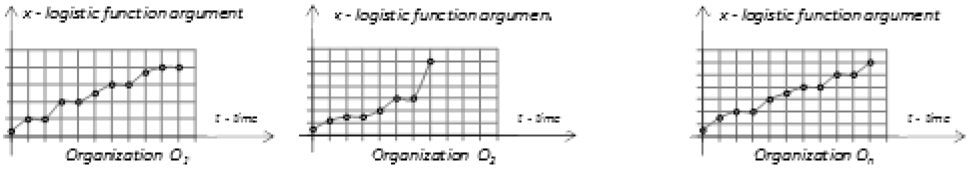

Fig. 7. Differences in the development rate of organizations.
Source: the authors.

The value of the argument (x) is determined(according to a properly designed algorithm) in the research process on the basis of the attributes measured for each organization (Figure 8), which allows us to indicate the level of technology (*PT)* using the model.

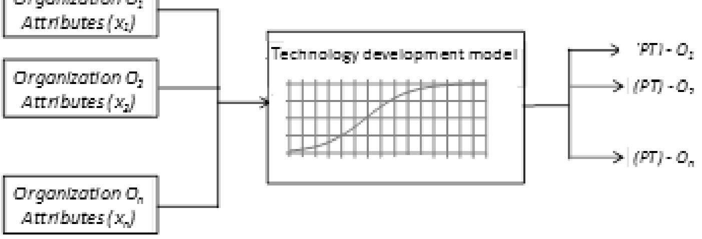

Fig. 8. Technology level of organizations.
Source: the authors.

4. Logistic function argument

In order to determine the argument measure of the logistic function constituting the model of technology development one can apply the centre of gravity method [6] with a growing tapering function. The technology level determined according to the presented model is the function of many partial characteristics (attributes) linked to the innovation level. Six levels of innovation can be assumed (similarly as in the technology level evaluation) with reference to partial assessments (β) conducted in the five-point Likert scale (Table 2). Partial assessments reflect how a particular technology affects the operation of the organization (0- slight influence, 1-influence with little significance, 2- significant influence, 3- highly significant, 4- dominant influence).

Table 2

Technologies within organisation		Technology level		Influence assessmentβ				
				0	1	2	3	4
Conservative	Definitely conservative	Z-a	β_1	x				
	Medium-conservative	Z-b	β_2			x		
	Moderately conservative	Z-c	β_3			x		
Innovative	Little innovative	I-a	β_4				x	
	Medium-innovative	I-b	β_5			x		
	Definitely innovative	I-c	β_6		x			

Source: the authors.

The interpretation of sample assessment indicated in Table 2 is the following: Medium-conservative (β_2= 2) and conservative (β_3= 2) technologies have a significant influence on the operation of an organization, little innovative technologies (β_4= 3) have a highly significant influence. Medium-innovative technologies have a significant influence (β_5= 2), and definitely innovative technologies have influence of little significance (β_6= 1). The presented partial assessments provide knowledge on the structure of innovation in the organization. They may be used for calculating the generalized value of the variable x, called further the innovation index:

$$x = x^{*}\varphi(x^{*}), \tag{12}$$

where the index $x*$ is calculated applying the centre of gravity method:

$$x^{*} = \frac{\sum_{i=1}^{6} i \cdot \beta_i}{\sum_{i=1}^{6} \beta_i}\,. \tag{13}$$

The tapering function *φ(x)* is to strengthen the influence of innovative technologies on the calculated index value. In the simplest case it can be the linear function (a, b-constant coefficients):

$$\varphi(x^{*}) = a \cdot x^{*} + b\ . \tag{14}$$

At the same time the function can be responsible for scaling the index x to the range of the applied logistic function.

The Likert scale accepted for the 6 levels allows for assigning each organization to 24 points. However, the application of the centre of gravity method enforces limiting this number in order to cover the whole range of changeability x with results. It has been determined in the conducted sample research that 8 points will be ascribed to each organization.

6. Sample results of the technology level diagnosis

Technology level evaluation according to the presented methodology requires the following actions concerning the examined organization:

- Identification of main areas to be assessed for the six modelled levels of innovation.
- Assignment of partial assessments according to the Likert scale.
- Computation of the innovation index value.
- Indication of values (PT) determining the level of technology.
- Determination of susceptibility to changes (PT).

The final stage is the juxtaposition and a graphical representation of results supplemented with a commentary in a descriptive form. In the presented examples the knowledge on organizations was gathered from public information sources, and not in a methodical research process. Therefore the names of the companies were not revealed, and the results should be treated with a limited trust, rather as an example of applying the methodology.

Organization O1

It is an manufacturer of various kinds of posts used mainly in road infrastructure - lighting, traffic lights, stadium lighting, etc. apart from classical posts with a simple structure ($\beta_2 = 1$) they produce posts with variable cross-section($\beta_3 = 3$), light-wall tubes with skeletal reinforcement ($\beta_4 = 2$), with the application of composites based on own patents ($\beta_5 = 1$). The company has its own Design and Research Unit ($\beta_6 = 1$).

Organization O2

It is a small manufacturer of dedicated electronic systems applied in the systems of automatics. Apart from classical systems ($\beta_3 = 1$) they produce complex systems with the application of the latest generation elements ($\beta_4 = 4$), systems with a large contribution of knowledge at the design stage ($\beta_5 = 2$). The company has a team searching for novelties in this field, which are systematically implemented into their solutions($\beta_6 = 1$).

Organization O3

It is a manufacturer of central heating boilers fired with solid fuels. Part of the production includes traditional coal-fired furnaces with simple automatic systems ($\beta_3 = 3$), and boilers with highly-efficient combustion and heat recovery systems equipped with advanced automatics containing electronic systems ($\beta_4 = 2$). Attempts are made to implement advanced solutions, own and those based on licenses, concerning the installation of advanced control systems in the boilers ($\beta_5 = 1$).

Organization O4

It is a wood plant producing garden fittings (pegs, hurdles, benches, arbours, pergolas, curtain panels). A large part of production involves simple, classical elements ($\beta_1 = 2$, $\beta_2 = 4$) made manually using simple machines and technologies. The plant has a wood drier ($\beta_3 = 1$) and a partially automated machine used for wood treatment ($\beta_4 = 1$). Proper data relating to partial assessment of the four organizations subject to research are presented in Table 3, whereas their graphic illustration is in Figure 8. Computations with the application of the model were conducted on the basis of the data. Table 4 contains the results.

Table 3

Innovation levels		Organization			
		O_1	O_2	O_3	O_4
Z-a	β_1	0	0	1	2
Z-b	β_2	1	0	1	4
Z-c	β_3	3	1	3	1
I-a	β_4	2	4	2	1
I-b	β_5	1	2	1	0
I-c	β_6	1	1	0	0

Source: the authors.

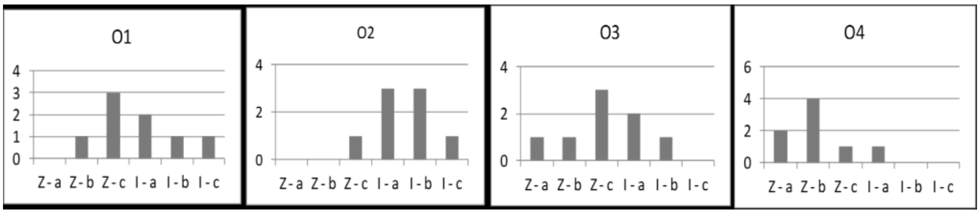

Fig. 9. Partial assessments of innovation.
Source: the authors.

Table 4

Organization	Innovation index x	Technology level (PT)	Innovation level	Sensitivity to changes (PT)'
O1	4,45	8,11	I-b	1,53
O2	5,89	9,48	I-c	0,50
O3	3,23	5,59	I-a	2,47
O4	1,67	2,09	Z-c	1,66

Source: the authors.

The analysis results in a graphic form are presented in Figure 10. The location of particular organizations is shown on the logistic curve (mathematical model). The position of organizations is also shown on a curve determining the susceptibility to changes (derivative of the logistic function), which in an indirect way diagnoses possibilities of development.
According to the analysis results (Table 4, Figure 9, 10) it is possible to present a descriptive diagnosis directed at technology level and a related innovation level in the examined organizations.

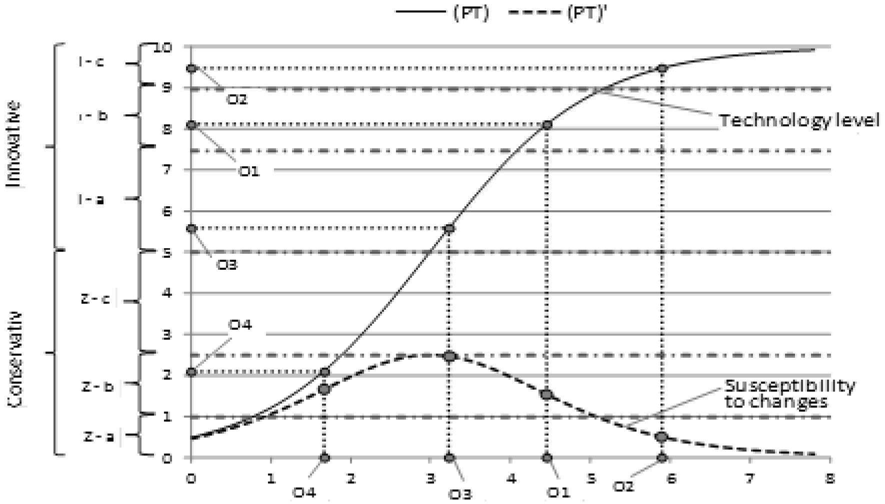

Fig. 10. Research results-graphical representation.
Source: the authors.

Organization O1

Medium-innovative technology level (in the central zone of the interval). High susceptibility to developmental changes (with a downward trend). Further development requires increased activity in the zone (I-c)-own Design and Research Unit.

Organization O2

Technology level definitely innovative (in the initial interval zone). A slight susceptibility to changes. It is indicated to intensify the search for own solutions (I-c) protected with patents.

Organization O3

Little-innovative technology level (in the final zone of the interval). Exceptionally high susceptibility to developmental changes, which gives a chance for development. Further development demands an increased activity in the zone(I-b)- implementation of modern solutions mainly based on buying licenses.

Organization O4

Medium-conservative technology level (in the central zone of the interval). High susceptibility to developmental changes (with an upward trend). Further development requires implementation of new products at a higher technological level, which is possible mainly through acquiring licenses and technology transfer.

7. Summary

Nowadays the innovation issues are frequently on research. Following this trend the paper presents an original approach to the issue of mathematical modelling and assessing innovation level primarily in manufacturing companies. An essential element seems to be the technology assessment (which is the measure of innovation) in the scale from a definitely conservative to a definitely innovative. It is an assessment of the company as a whole with the account of the significance of partial innovative activities. This approach can determine the company condition and its possibilities in a more authentic way and as the result present suggestions for development activities.

The presented modelling methodology and evaluation solves part of the problems connected with constructing a diagnostic tool as a map in the form of a matrix of states defined by two parameters: technology level and innovative environment development [2, 3, 4]. The application of the recommended methodology requires still a lot of research work directed at indicating credible parameters of the modelled logistic curve and inventing a technique of gathering data on the examined companies. It will allow us to conduct research of a wider range than it has been presented in the examples treated only as a methodology presentation.

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