OPTIMAL DECISION MAKING IN COMPETITION CONDITIONS OF A PRODUCTION ORGANISATION PROCESS EMPLOYING GAME THEORY

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Abstract: Market competition of a production organisation process and a problem of optimal decision making on the market are presented in the paper. The model of the problem is based on game theory. It is a non-cooperative two-person game in which competing production enterprises are players and points assessing enterprises products are payoffs. The solution of the game, identified by the best Nash equilibrium, indicates optimal decisions that the competing enterprises should take in order to enter or stay in the market. The game is illustrated by numerical examples to show the best choices among the obtained pure Nash equilibria. One is a payoff dominance strategy profile and the other is a risk dominance strategy profile.

Key words: game theory, Nash equilibrium, production organisation

1. Introduction

Many enterprises use various resources in order to gain a support in the activities connected with placing their products on the market. The origin and type of the resources can refer to each stage of the activities, starting from a design process, then realisation of a production process, and finally the delivery of a ready product to a consumer. The support can be provided in the following areas: knowledge (know-how), management, materials, tools and machinery, finances, marketing and promotion, contacts with customers. In many cases, an enterprise accepts support offers in return for a profit share in the firm that is the support offer provider. The form of the profit is usually agreed on in a form accepted by both business parties.

Profits do not have to be financial or of one type only. Being a monopolist, the firm which provides the support can define the support range as well as the support criteria, allowing the possibility of cooperation with more than one enterprise. Therefore, the enterprises seeking the support are competitors. They will act to fulfil the criteria of the supporting firm so as to maximize its own chance for the cooperation, regardless of what other enterprises do.

The above-mentioned problem of optimal decision making can be formulated and solved using game theory.

2. Problem formulation on the basis of game theory

Game is a conflict or a cooperation situation in which at least one of the entrants behaves so that they could maximise their own benefits by predicting other entrants' reaction. The situation can be described in the language of game theory if the following requirements are met [1, 2, 3]:

1. There have to be at least two game entrants – players of the game.
2. Each player has to choose between at least two actions that are responses to the situation, the action is called a strategy.
3. Each player receives a payoff for each game outcome; the payoff depends on the strategies chosen by all the players.
4. All the conditions of the game are known by the players.

The payoff (profits or losses) is defined as a numerical value attributed to the player. It depends on the current strategy choices of all the players (strategy profile). The product of the numbers of individual players strategies specifies both the number of strategy profiles and the number of possible payoffs to each player. The decisions taken by all the players affect the final outcome of the game.

Game theory can be applied in the situation of a competition in a production organization market as it is described below.

There are enterprises looking for a support in their production organisation process. A firm declares such a support. However, in order to obtain the support some criteria have to be met: to make the support cooperation begin, a competing product has to have m characteristics out of the expected n features defined by the supporting firm. Out of the m characteristics, the product is assessed on the basis of $k < m$ features that have to be indicated as the most vital according to the applicant. The supporting firm assigns some points to each feature to evaluate the product better: the more highly valued a feature is, the greater number of points it gets. The total sum of the points is used for the final assessment which affects the range of the provided support.

The supporting firm decides on the algorithm of calculating the assessment, which depends on the firm market policy. Each competing enterprise should indicate the set of the product characteristics that could obtain a maximally good assessment, taking into account the decisions of other enterprises.

The situation, elaborated by using game theory [4], is called *Enterprise game* further on.

1. Players

   Production enterprises are players. They seek a support for their production organisation process. The players are to obtain the best possible assessment for their products.
2. Strategies

   A strategy is defined by the set of k features which the enterprise will indicate out of m characteristics of their product as the most recommended. Therefore, the number of the strategies for the player is determined by k-element combinations without replication of the m-element set. The value is expressed by the Newton symbol:

   $$\binom{m}{k} = \frac{m!}{k! \cdot (m-k)!} \tag{1}$$
3. Payoff to players

   The total sum of the points assigned to the product for the indicated k features is used by the supporting firm to define a payoff for each strategy of the players.

   If the payoff is calculated by summing all the points assigned to the features in a strategy, each player will choose their best strategy indicating the set of features that provides the maximum number of the total points. The solution is straightforward and game theory is not necessary to describe the problem.

   A different situation arises when the supporting firm introduces a weight for each product assessment, indicating the significance of the assessment in relation to other products. The weight value can depend on how widespread a feature is among all the products. For a certain strategy, the more (the less) frequently the feature occurs in a strategy profile it belongs to, the greater (or the smaller) the weight is.

4. Game conditions

Some assumptions, presented below, are made in order to provide more precise game conditions.

There are two competing production enterprises A and B. A supporting firm F declares the number of the product characteristics *n* equal to 7, the number of the required features *m* equal to 5, and the number of the recommended features *k* equal to 3.

The product features and points assigned to each feature are presented in table 1. The features that are specific for the products of enterprises A and B are marked with a plus sign (+).

Table 1. Product features and their relation to products A and B

Feature description	**Feature symbol**	**Points for a feature**	**Relation to product A**	**Relation to product B**
Characteristic *a*	a	o_a		+
Characteristic *b*	b	o_b	+	+
Characteristic *h*	h	o_h		+
Characteristic *d*	d	o_d	+	
Characteristic *e*	e	o_e	+	+
Characteristic *f*	f	o_f	+	
Characteristic *g*	g	o_g	+	+

Each player has ten strategies to choose from. It is calculated using the formula (1) for the values $m = 5$ and $k = 3$:

$$\binom{5}{3} = \frac{5!}{3! \cdot 2!} = 10 \quad (2)$$

Table 2 represents the matrix form of *Enterprise game*. The matrix column header contains all possible strategies for player A. The matrix row header contains all possible strategies for player B. The payoffs are shown in the cells. The first number in the table cell is the payoff received by the row player (player A), whereas the second one is the payoff to the column player (player B). The total points for the features creating the strategies of players A and B are given in the last column and the last row of the matrix respectively.

The payoff $A_{i,j}$ to player A depends on their strategy Ai and the strategy Bj of player B, and it is expressed by the following formula:

$$A_{i,j} = O_{\mathrm{A}i} \cdot w_{i,j} \quad (3)$$

The payoff $B_{i,j}$ to player B depends on their strategy Bj and the strategy Ai of player A, and it is expressed by the following formula:

$$B_{i,j} = O_{\mathrm{B}j} \cdot w_{ij} \quad (4)$$

In relationships (3) i (4), $O_{\mathrm{A}i}$ and $O_{\mathrm{B}j}$ are the sums of the points for strategy Ai of player A, and for strategy Bj of player B respectively (e.g.: $O_{\mathrm{A}1}=o_b+o_d+o_e$, $O_{\mathrm{B}10}=o_h+o_e+o_g$).

The values $w_{i,j}$ are the weights which increase (or decrease) the totals and describe the payoffs depending on how many common features of a product there are in strategies A*i* and B*j* of both players.

Table 2. Enterprise game matrix

Player B / Player A	**B1:** abh	**B2:** abe	**...** ...	**B5:** ahg	**...** ...	**B9:** beg	**B10:** heg	Total points for player A
A1: bde	A(1,1), B(1,1)	…	…	…	…	…	…	O_{A1}
A2: bdf	…	…	…	…	…	…	…	O_{A2}
A3: bdg	…	…	…	…	…	…	…	O_{A3}
A4: bef	…	…	…	…	…	…	…	O_{A4}
A5: beg	…	…	…	…	…	…	…	O_{A5}
A6: bfg	…	…	…	…	…	…	…	O_{A6}
A7: def	…	…	…	A(7,5), B(7,5)	…	…	…	O_{A7}
A8: deg	…	…	…	…	…	…	…	O_{A8}
A9: dfg	…	…	…	…	…	…	…	O_{A9}
A10: efg	…	…	…	…	…	…	A(10,10), B(10,10)	O_{A10}
Total points for player B	O_{B1}	O_{B2}	…	O_{B5}	…	O_{B9}	O_{B10}	

Each player should strive to obtain the best results, making decisions independently, without looking at the decisions of the other player. This is a non-cooperative game in which everybody benefits [1, 5]. The fundamentals for optimal decisions of the players in such a game is a concept of constituting the equilibrium.

In game theory, the equilibrium is defined by a profile of the strategies that are the best responses for one another [6]. It is called a Nash equilibrium – the set of strategies in which no player can increase the payoff by unilaterally changing their own strategy. The optimal decision of a player can be expressed as follows: “I will do what is the best for me while you are doing what you are doing”.

A Nash equilibrium for *Enterprise game* is such a profile in pure strategies in which both production enterprises will gain as much support as possible. Consequently, enterprises A and B will be in the Nash equilibrium if each of them is making the best decision it can, taking into account the decisions of the other enterprise. The decision is the set of three characteristics of the product presented to firm F, knowing the scoring of the features and the weights w_{ij}.

The Nash equilibria for a two-person matrix game can be identified according to the following rules:

- in each row (i.e. for each strategy of player A) the strategy that gives their highest payoff has to be chosen for player B – the strategy has to be marked in a matrix game; this is the best response of player B for a given strategy of player A;

- in each column (i.e. for each strategy of player B) the strategy that gives their highest payoff has to be chosen for player A – the strategy has to be marked in a matrix game; this is the best response of player A for a given strategy of player B;
- if both, the first payoff value and the second payoff value, are marked in one cell, then the cell represents a Nash equilibrium.

3. Optimal choice of product features

In each non-cooperative game there can be more than one Nash equilibrium. Then, in the case of pure strategies, the best equilibrium is indicated according to the concept by J. Harsanyi and R. Selten [6]:

- among all the Nash equilibria, players should choose the strategy profile the payoff of which dominates the other payoffs;
- if the payoff dominance does not exist, the players should choose the strategy profile that is risk dominant, i.e. the profile that has the largest basis of attraction, which means – is less risky.

To better understand the concept, let us consider a game matrix such as the one presented in table 3. The cells that indicate the Nash equilibria have grey background.

Table 3. Two-person game matrix with two Nash equilibria

Player B / Player A		**Bj**	**Bl**
		1 – q	q
Ai	1 – p	$A_{i,j}$; $B_{i,j}$	$A_{i,l}$; $B_{i,l}$
Ak	p	$A_{k,j}$; $B_{k,j}$	$A_{k,l}$; $B_{k,l}$

A strategy pair (A*i*, B*l*) payoff dominates a strategy pair (A*k*, B*j*) if the following conditions are satisfied for the respective payoffs: $A_{i,l} \geq A_{k,j}$ and $B_{i,l} \geq B_{k,j}$ and at least one of the two is a strict inequality: $A_{i,l} > A_{k,j}$ or $B_{i,l} > B_{k,j}$.

Now, suppose neither the strategy pair (A*i*, B*l*) nor the strategy pair (A*k*, B*j*) payoff dominates each other, i.e. (case 1): $A_{i,l} \geq A_{k,j}$ and $B_{i,l} \leq B_{k,j}$ and at least one of the two is a strict inequality or (case 2) $A_{i,l} \leq A_{k,j}$ and $B_{i,l} \geq B_{k,j}$ and at least one of the two is a strict inequality.

In order to indicate the risk dominant equilibrium, risk factors for all the equilibria have to be calculated. The equilibrium with the lowest risk factor is the risk dominant equilibrium [6, 7].

In the paper, the risk dominant quilibrium is found for the case 2 of no payoff dominance game; *p* denotes probability of choosing a better strategy A*k* by player A, whereas *q* denotes probability of choosing a better strategy B*l* by player B.

The expected payoffs to player A have to be calculated for the strategies A*i* and A*k* in order to determine the risk factor for player A:

$$\begin{aligned} E(payoff(Ai)) &= (1-q) \cdot A_{i,j} + q \cdot A_{i,l} \\ E(payoff(Ak)) &= (1-q) \cdot A_{k,j} + q \cdot A_{k,l} \end{aligned} \tag{5}$$

Player A will choose their better strategy A*k* if the expected payoff for the strategy is higher than the expected payoff for the worse strategy A*i*:

$$(1-q) \cdot A_{k,j} + q \cdot A_{k,l} > (1-q) \cdot A_{i,j} + q \cdot A_{i,l} \tag{6}$$

The solution $q < q_0$ of the inequality (6) delivers the quantity q_0 which is the risk factor for the equilibrium (A*k*, B*j*).

To determine the risk factor for player B, the expected payoffs to the player have to be calculated for their strategies B*j* and B*l*:

$$\begin{aligned} E(payoff(Bj)) &= (1-p) \cdot B_{i,j} + p \cdot B_{k,j} \\ E(payoff(Bl)) &= (1-p) \cdot B_{i,l} + p \cdot B_{k,l} \end{aligned} \tag{7}$$

Player B will choose their better strategy B*l* if the expected payoff for this strategy is higher than the expected payoff for the worse strategy B*j*:

$$(1-p) \cdot B_{i,l} + p \cdot B_{k,l} > (1-p) \cdot B_{i,j} + p \cdot B_{k,j} \tag{8}$$

The solution $p < p_0$ of the inequality (8) delivers the quantity p_0 which is the risk factor for the equilibrium (A*i*, B*l*).

The lower of the two values p_0 and q_0 indicates the risk dominance of the Nash equilibrium. It implies that the more uncertain one player is about the other player's action, the more likely they are to choose the strategy corresponding to the action.

3.1. Choosing an optimal set of product features by payoff dominance

Let us consider the situation in which the market policy of firm F does not prefer the uniqueness of features. Consequently, the weight values $w_{i,j}$ will decrease as the number of common features occurring in products A and B will decrease.

Firm F sets the conditions of the product assessment establishing the conditions of the game, named here GF1. The list of points assigned to the product features is shown in table 4. The weight equal to 1 is assumed to be the highest value for the maximum number of the common features in both strategies, i.e. for 3 features. When the weight value decreases by 10% as the number of common features decreases by 1, the following values of $w_{i,j}$ are used to modify the total points for a product: 0.9 when the number of common features is 2, 0.8 if it is 1, and 0.7 if there are no common features in any strategy.

Table 4. Points for product features in game GF1

a	**b**	**h**	**d**	**e**	**f**	**g**
10	6	7	8	10	9	7

The matrix form of game GF1 is shown in table 5. Each cell contains a payoff calculated for both products A and B , taking into account the weight values. For example, the total points for strategy A1 = *bde* is equal to 24. However, the payoffs $A_{1,j}$ depend on the strategies *j* of player B, and when $B1$ = *abh* then $A_{1,1}$ is equal to 19.2 (= 24·0.8; one feature is common), when B2 = *abe* then $A_{1,2}$ is equal to 21.6 (= 24·0.9; two features are common), and so on.

Table 5. Matrix for game GF1 with optimal Nash equilibrium by payoff dominance

Player B / Player A	B1: abh	B2: abe	B3: abg	B4: ahe	B5: ahg	B6: aeg	B7: bhe	B8: bhg	B9: beg	B10: heg	Total points for A	Maximum payoff for B
A1: bde	19.2; 18.4	21.6; 23.4	19.2; 18.4	19.2; 21.6	16.8; 16.8	19.2; 21.6	21.6; 20.7	19.2; 16	21.6; 20.7	19.2; 19.2	24	23.4
A2: bdf	18.4; 18.4	18.4; 20.8	18.4; 18.4	16.1; 18.9	16.1; 16.8	16.1; 18.9	18.4; 18.4	18.4; 16	18.4; 18.4	16.1; 16.8	23	20.8
A3: bdg	16.8; 18.4	16.8; 20.8	18.9; 20.7	14.7; 18.9	16.8; 19.2	16.8; 21.6	16.8; 18.4	18.9; 18	18.9; 20.7	16.8; 19.2	21	21.6
A4: bef	20; 18.4	22.5; 23.4	20; 18.4	20; 21.6	17.5; 16.8	20; 21.6	22.5; 20.7	20; 16	22.5; 20.7	20; 19.2	25	23.4
A5: beg	18.4; 18.4	20.7; 23.4	20.7; 20.7	18.4; 21.6	18.4; 19.2	20.7; 24.3	20.7; 20.7	20.7; 18	23; 23	20.7; 21.6	23	24.3
A6: bfg	17.6; 18.4	17.6; 20.8	19.8; 20.7	15.4; 18.9	17.6; 19.2	17.6; 21.6	17.6; 18.4	19.8; 18	19.8; 20.7	17.6; 19.2	22	21.6
A7: def	18.9; 16.1	21.6; 20.8	18.9; 16.1	21.6; 21.6	18.9; 16.8	21.6; 21.6	21.6; 18.4	18.9; 14	21.6; 18.4	21.6; 19.2	27	21.6
A8: deg	17.5; 16.1	20; 20.8	20; 18.4	20; 21.6	20; 19.2	22.5; 24.3	20; 18.4	20; 16	22.5; 20.7	22.5; 21.6	25	24.3
A9: dfg	16.8; 16.1	16.8; 18.2	19.2; 18.4	16.8; 18.9	19.2; 19.2	19.2; 21.6	16.8; 16.1	19.2; 16	19.2; 18.4	19.2; 19.2	24	21.6
A10: efg	18.2; 16.1	20.8; 20.8	20.8; 18.4	20.8; 21.6	20.8; 19.2	23.4; 24.3	20.8; 18.4	20.8; 16	23.4; 20.7	23.4; 21.6	26	24.3
Total points for B	23	26	23	27	24	27	23	20	23	24		
Maximum payoff for A	20	22.5	20.8	21.6	20.8	23.4	22.5	20.8	23.4	23.4		

A single frame is used to mark the best responses of player B for consecutive strategies of player A – these are the highest payoffs of player B in each row. A double frame is used to mark the best responses of player A for consecutive strategies of player B – these are the highest payoffs of player A in each column.

The payoffs for all the Nash equilibria in pure strategies are indicated by grey cells: (A4, B2) $\rightarrow$ ($A_{4,2}$; $B_{4,2}$) = (22.5; 23.4), (A7, B4) $\rightarrow$ ($A_{7,4}$; $B_{7,4}$) = (21.6; 21.6), and (A10, B6) $\rightarrow$ ($A_{10,6}$, $B_{10,6}$) = (23.4; 24.3). Among them, the payoff dominance equilibrium is the strategy profile (A10, B6) – the payoffs $A_{10,6}$ and $B_{10,6}$ are higher than the respective payoffs in other Nash equilibria. Therefore, enterprises A and B should declare the following features of their products as the most recommended: *efg* (strategy A10) i *aeg* (strategy B8) respectively. Thus, the support in the production organisation process is determined by 23.4 points assigned to product A and 24.3 points assigned to product B.

3.2. Choosing an optimal set of product features by risk dominance

Now, consider the case when the market policy of firm F is opposite to the one described in the previous section. The weight values $w_{i,j}$ will decrease as the number of common features for products A and B will increase.

Firm F sets the conditions of the product assessment establishing the conditions of a new game, named here GF2.The list of points assigned to the product features is shown in table 6. The weights $w_{i,j}$ are strongly discriminating and they decrease by half when the number of common features increases by one. The weight equal to 1 is assumed to be the highest value for the minimum number 0 of common features in both strategies. Then the following values of $w_{i,j}$ are used to modify the total points for a product: 0.5 when the number of common features is 1, 0.25 if it is 2, and 0.125 if all the features are common in both strategies.

Table 6. Points for product features in game GF2

a	b	h	d	e	f	g
10	6	7	8	10	7	8

The matrix form of game GF2 is shown in table 7. Each cell contains the payoff calculated for both products A and B taking into account the weight values. For example, the total points for strategy A1= *bde* is equal to 24. However, payoffs $A_{1,j}$ depend on the strategies j of player B, and when $B1$ = *abh* then $A_{1,1}$ is equal to 12 (= 24•0.5; one feature is common), when B2 = *abe* then $A_{1,2}$ is equal to 6 (= 24•0.25; two features are common), and so on.

All Nash equlibria in pure strategies are indicated by grey cells for their payoffs: (A3, B6) → ($A_{3,6}$; $B_{3,6}$) = (21; 28), (A7, B1) → ($A_{7,1}$; $B_{7,1}$) = (26; 23), (A8, B5) → ($A_{8,5}$; $B_{8,5}$) = (25; 25), and (A9, B4) → ($A_{9,4}$, $B_{9,4}$) = (23; 27). Out of them, the best payoff choice for player A is strategy A7 = *def* and the best payoff choice for player B is strategy B6 = *aef*. Both strategies belong to different Nash equilibria. Therefore, there is no payoff dominance equilibrium. A risk dominance equilibrium has to be indicted to choose the optimal strategies for both players.

Table 7. Matrix for game GF2 with optimal Nash equilibrium by risk dominance

Player B / Player A	**B1:** abh	**B2:** abe	**B3:** abg	**B4:** ahe	**B5:** ahg	**B6:** aeg	**B7:** bhe	**B8:** bhg	**B9:** beg	**B10:** heg	Total points for A	Maximum payoff for B
A1: bde	12; 11.5	6; 6.5	12; 12	12; 13.5	24; 25	12; 14	6; 5.8	12; 10.5	6; 6	12; 12.5	24	25
A2: bdf	10.5; 11.5	10.5; 13	10.5; 12	21; 27	21; 25	21; 28	10.5; 11.5	10.5; 10.5	10.5; 12	21; 25	21	28
A3: bdg	11; 11.5	11; 13	5.5; 6	22; 27	11; 12.5	11; 14	11; 11.5	5.5; 5.3	5.5; 6	11; 12.5	22	27
A4: bef	11.5; 11.5	5.8; 6.5	11.5; 12	11.5; 13.5	23; 25	11.5; 14	5.8; 5.8	11.5; 10.5	5.8; 6	11.5; 12.5	23	25
A5: beg	12; 11.5	6; 6.5	6; 6	12; 13.5	12; 12.5	6; 7	6; 5.8	6; 5.3	3; 3	6; 6.3	24	13.5
A6: bfg	10.5; 11.5	10.5; 13	5.3; 6	21; 27	10.5; 12.5	10.5; 14	10.5; 11.5	5.3; 5.3	5.3; 6	10.5; 12.5	21	27
A7: def	25; 23	12.5; 13	25; 24	12.5; 13.5	25; 25	12.5; 14	12.5; 11.5	25; 21	12.5; 12	12.5; 12.5	25	25
A8: deg	26; 23	13; 13	13; 12	13; 13.5	13; 12.5	6.5; 7	13; 11.5	13; 10.5	6.5; 6	6.5; 6.3	26	23
A9: dfg	23; 23	23; 26	11.5; 12	23; 27	11.5; 12.5	11.5; 14	23; 23	11.5; 10.5	11.5; 12	11.5; 12.5	23	27
A10: efg	25; 23	12.5; 13	12.5; 12	12.5; 13.5	12.5; 12.5	6.3; 7	12.5; 11.5	12.5; 10.5	6.3; 6	6.3; 6.3	25	23
Total points for B	23	26	24	27	25	28	23	21	24	25		
Maximum payoff for A	26	23	25	23	25	21	23	25	12.5	21		

The matrix for game GF2 is reduced to a two-person game – as presented in table 8. The two Nash equilibria from table 7 of game GF2 are taken: (A2, B6) – with the best payoff to player B and (A8, B1) – with the best payoff to player A. Additional strategy profiles that complete the reduced matrix are the following: (A2, B1) – the profile indicated by the row with the best strategy of player B and the column with the best strategy of player A, and (A8, B6) – the profile indicated by the row with the best strategy of player A and the column with the best strategy of player B. The probability of choosing a better strategy by player A is denoted by p and the probability of choosing a better strategy by player B by q.

Table 8. Reduced matrix game for game GF2

Player B / Player A		B1: abh	B6: aeg
		1 - q	q
A2: bdf	1 - p	10,5; 11,5	21; 28
A8: deg	p	26; 23	6,5; 7

Player A will choose their better strategy A8 if the following inequality holds:

$$26 \cdot (1-q) + 6.5 \cdot q > 10.5 \cdot (1-q) + 21 \cdot q \Rightarrow \boldsymbol{q < q_0 = 0.517} \quad (9)$$

Player A will choose their better strategy if they expect that the probability q of playing the better strategy by player B is less than q_0= 0.517.

Player B will choose their better strategy B6 if the following inequality holds:

$$28 \cdot (1-p) + 7 \cdot p > 11.5 \cdot (1-p) + 23 \cdot p \Rightarrow \boldsymbol{p < p_0 = 0.508} \quad (10)$$

Player B will choose their better strategy if they expect that the probability p of playing the better strategy by player A is less than p_0= 0.508.

It results from the relationship p_0= 0.508 < q_0= 0.517. Therefore, both players should choose the Nash equilibrium which contains a better strategy for player A. Player A will have stronger arguments than player B to choose a better strategy. Therefore, enterprises A and B should declare the following features of their products as the most recommended: *deg* (strategy A8) i *abh* (strategy B1). Thus, the support in the production organisation process is determined by 26 points assigned to product A and 23 points assigned to product B.

4. Summary

Game theory is a method of applied mathematics. It has been widely recognized as an important tool in many fields. The most popular are: economics, political science, psychology, logic, computer science, and biology. However, it can also be applied in engineering sciences, when optimal decisions have to be taken.

The paper presents the application of game theory for possible behaviours of some production enterprises in a decision-making process. The enterprises are assumed to be players in a non-cooperative game. Their competition situation refers to the organisation of a production process and to the need to obtain external support to carry out the activity. Each player has to select the strategy that allows him to gain as much support as possible in the conditions defined by an external firm offering the support. It requires a bargaining solution such as a Nash equilibrium.

Numerical examples illustrate the situation for different game conditions. There are more pure Nash equilibria than one in each example. Therefore, payoff dominance and risk dominance strategy profiles are suggested as optimal decision choices. However, it should be mentioned that pure Nash equilibria are not always possible, which requires other approaches to find optimal solutions.

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